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Subway Station Real-time Indoor Positioning System for Cell Phones

Chengqi Ma[1]
[1]Department of Electronic & Electrical Engineering
University College London, UCL
London, United Kingdom WC1E 7JE
chengqi.ma.16@ucl.ac.uk

Chenyang Wan[1]; Yuen Wun Chau[1]; Soong Moon Kang[2]; David R. Selviah[1];
[2]School of Management
University College London, UCL
London, United Kingdom WC1E 7JE

***Abstract*—As wireless local area network, WLAN, access point (AP) are becoming very common wireless communication infrastructures in indoor environments, Wi-Fi signal based Indoor Positioning Systems (IPS) have been widely developed in recent years and one of the most popular technologies is the received signal strength (RSS) fingerprinting technology. However, due to large amount of time-consuming work required for offline calibration in large indoor environments, researchers have investigated generating the calibration database while walking about instead of carrying out measurements over a time period at fixed reference points [1]. This paper combines both Wi-Fi fingerprinting and Pedestrian Dead-reckoning (PDR) technologies to introduce a real-time indoor navigation system for large complex three-dimensional indoor environments including a novel calibration method with associated novel matching algorithms. Detailed experiments were conducted in two subway stations with complicated structure under normal operating conditions in which trains regularly arrived and departed and groups of people walked to and from the trains. The results for real cell phone tracking on phones carried by passengers, give a satisfactory error of 2.9 metres during peak congestion times and 1.7 metres when few people were in the station.**

Keywords*—Wi-Fi fingerprinting; Pedestrian Dead-reckoning (PDR), indoor positioning system (IPS); Kalman Filter; Power strength histogram, subway station, signal processing.

I. INTRODUCTION

More than half of the world’s population is now living in large urban centres, and the number of cities with over 10 million inhabitants is also increasing [2], imposing a major challenge for urban transportation systems. In this context, electric subway trains beneath the ground surface are viewed as an effective way to deal with congestion and environmental pollution that these megacities generate. For the safety and comfort of subway passengers, it is important to understand the patterns of passenger behaviour, including their movements through mostly overcrowded and complex subway stations.

The recent proliferation of Wi-Fi-equipped smartphones and the rapid expansion of Wi-Fi zones in most subway stations provides an unprecedented opportunity to understand passenger behaviour and movements that can help subway operators and planners to offer a better service.

In this study, in order to achieve the aim of public service improvement, we introduce a real-time Wi-Fi fingerprinting and pedestrian dead reckoning (PDR) hybrid technology based IPS for subway stations with a novel method for offline calibration method designed for use in large-scale (larger than 20,000 m^3) indoor environments. Previous work [1], [3] in similar scale environments achieved a positioning error distance of around 6 metres; however, our system is capable of providing a more accurate positioning service.

II. BACKGROUND REVIEW

Wi-Fi Fingerprinting and Pedestrian Dead-reckoning are two commonly used techniques for indoor positioning due to their low demand in extra infrastructure for deployment.

A. Wi-Fi fingerprinting technology

Within the scope of indoor positioning for smartphones, radio frequency fingerprinting is currently considered the most accurate technique [4]. A radio fingerprint is the pattern of radio signal strength measurements that is observed at a given location. It comprises a vector of signal identity information (e.g. Wi-Fi MAC addresses, or cellular Cell-IDs) and a corresponding vector of values of Received Signal Strength (RSS).

Wi-Fi measurements offer advantages compared to cellular phone signals and magnetic field measurements on smartphones, which are two other common Fingerprinting positioning techniques. As opposed to cellular signals, Wi-Fi signals offer greater dynamic ranges as they use short-range transmitters and Wi-Fi infrastructure has a better presence inside buildings in general [5]. Hence, Wi-Fi signals show more anisotropy than the external cellular signal sources, which are generally distant from each other. Magnetic field measurements produced by magnetometers provide a high dynamic range on a very fine scale [5] but can only provide a single contribution to the fingerprint vectors, which are outperformed by the many Wi-Fi measurements available in typical metropolitan indoor environments.

The Wi-Fi fingerprinting technique conventionally consists of two phases: an off-line calibration phase and an on-line testing phase. The calibration phase describes the process of collecting on smartphones over a certain time period, the RSS at different locations in a given space. The RSS data collected is then used to establish a radio map in a database indicating the average RSS measurements at each of the surveying locations. These average measurements, which are normally described in a vector manner, are the fingerprints. During the on-line testing phase, real-time RSS measurements collected on a smartphone are matched with the fingerprints in the established database. Localisation of the smartphone can then be achieved when a fingerprint location with the closest value of RSS vector is found.

B. Pedestrian Dead-reckoning (PDR)

PDR refers to the technique that tracks smartphones' positions using the phone's built-in inertial sensors [6]. Unlike most of the other localisation techniques, PDR only provides relative estimates of position and rotation, instead of absolute positions (i.e. the change in position since the last update). In addition, drift errors [7] can accrue quickly without the support of an external reference. As a result, many most recent solutions combine PDR with other localisation techniques [8] such as Wi-Fi fingerprinting which can serve as a source of absolute initial positions and position updates whenever the PDR errors reach a threshold value.

To date, mainly two types [7] of PDR techniques have been deployed in localisation systems: Inertial Navigation System (INS) and Step-and-Heading System (SHS). An INS is a system that tracks positions of smartphones via estimating full 3-D trajectories based on the sensor data. An SHS is applied specifically to the tracking of smartphones held by pedestrians. It estimates positions by accumulating vectors representing steps of the users.

III. System Implementation

This section introduces our whole system based on combining Wi-Fi fingerprinting with PDR.

A. Calibration and Mapping

In the traditional fingerprinting offline calibration phase, the RSS measurement and establishment of a fingerprint database is conducted over a certain time period at fixed reference points. However, for large and complex environments, this method can be extremely labour-intensive and time-consuming and can seriously impede and can block passenger movement during the measurement. Manual measurements and recordings at reference point coordinates can be error prone. Furthermore, it is difficult to ensure that there is a similar situation when collecting data at different reference points, which can lead to large errors at the online matching phase.

In order to overcome these issues, we designed a novel method to carry out calibration while walking around. First, the station is divided into several areas with simple structures e.g. platform, escalator and corridor without corner, into "sub-areas", and the calibration work is conducting by walking back and forth within the same sub-area several times taking automatic readings of signal strength every fraction of a second. Then, each sub-area is simplified into a series of uniformly distributed reference points and saved as a sub-area database. Also, while the calibration work is being carried out, simultaneous mapping work is carried out using the PDR technology by also taking recordings from those sensors every fraction of a second.

1) Body block effect

Since a person's body who is holding the phone, is close to the phone antenna it may possibly have a large effect on signals receiving from all the APs behind the person. The RSS can be very different when the phone holder is walking in different directions. Fig. 1 is an example of the RSS distribution on a straight platform received from one AP when the device holder walks back and forth for ten times. The red line is for when the person is walking in one direction and the blue line is for when the person has turned around and is walking back in the opposite direction with the phone held in front of them.

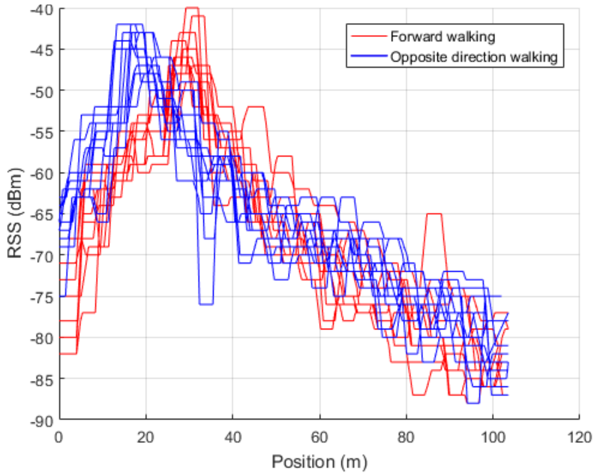

Fig. 1. Two opposite directions walking RSS distribution comparison

It can be observed that the RSS peak detected positions (relatively close to the AP) corresponding to walking in two opposite directions are of similar magnitude but are some distance away from each other. Hence, in our system, walking in different directions in the same area is recorded into a separate database with recorded direction information from the PDR.

2) Environmental variation effect

Since our experimental environment is a subway station, the Wi-Fi signal strength can be significantly influenced by periodically arriving and departing metallic trains as well as groups of people getting on and off the trains and walking around the station itself. We carried out field experiments in subway stations under normal operating conditions to observe that for a few people situation with the train and groups of people situations, the average RSS variation at a certain position can increase from 5 dB to 15 dB.

In order to cover all environmental situations, we carried out calibration while walking during different time periods to experience as much variation as possible in the environmental conditions to give a robust database which could deal with all these service conditions.

3) Database establishment

After data collection work, the walking trajectory of each sub-area is uniformly simplified into a series of reference points by resolution grid, δd , and the centre of each δd represents the position of a reference point. Then the RSS dataset of one sub-area is divided into different data groups, which means that all data collected within the same δd by multiple walking are put into the same group. Then, each data group is classified by MAC address (classified by transmitter AP). Afterwards, all data belonging to one AP at one reference point is processed by RSS histogram distribution, and each distribution is considered as one feature histogram distribution of a fingerprint. Fig. 2 are four examples of RSS distributions probability density function (PDF) from one AP at different positions.

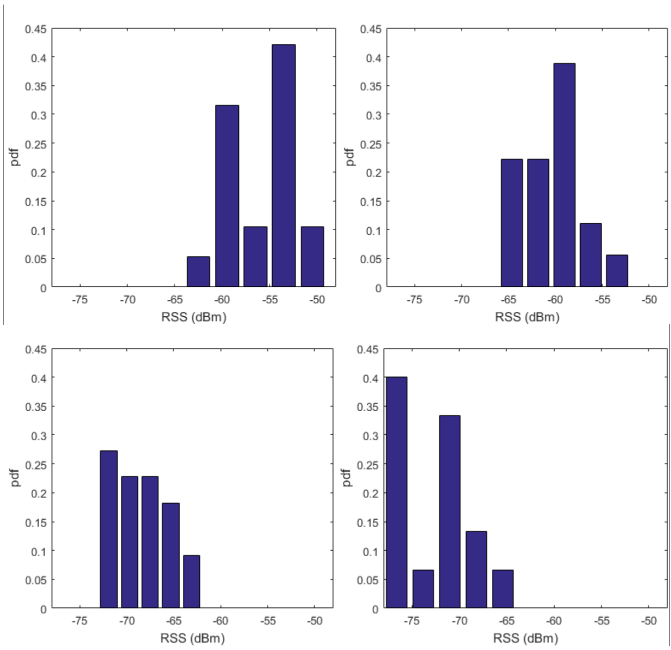

Fig. 2. RSS histogram distributions from one AP at different reference points.

We found that due to this database establishment method and body block and environmental variation effects we mentioned before, the RSS (from one AP) histogram distribution at one simplified reference point is no longer fitted with any standard distribution model [9] and are quite different from each other. Hence, we record all unique normalised RSS histogram distributions, which we call feature distributions, corresponding with the coordinate of reference point and AP information as a radio fingerprint.

In our experimental environment, the number of detectable AP in two subway stations was 1104 in total and at certain positions, the number of detectable AP was around 40, which means that each reference point has about 40 unique features for matching.

When the resolution δd and the number of bins k are changed, the system performance can be very different. Here we recommend to set these two parameters to ensure each distribution width is close to 10 dB and each bin width is about, BW= 2 dB. In our case, we set $\delta d = 1$ metre.

Other than unique distribution and direction information, we record an extra information of each fingerprint feature called the probability of occurrence, R , where

$$R = \frac{Number\ of\ collected\ datapoint\ from\ current\ AP}{Maximum\ number\ of\ collected\ datapoint\ in\ all\ APs}$$

In our work, we have simplified 2382 reference points in two stations. In the traditional calibration method, if we measured the RSS data at each fixed position for three minutes, then it would take nearly 120 hours (two weeks' work) to finish the calibration work, and the database of reference points can only include the environmental situation during the recording three minutes. In our method, in two weeks, even though we cannot collect as many data points for each reference point as in the traditional method, our database is capable of recording almost all situations occurring in the station. Another advantage is that our database is extendable, which means that every dataset collected by a passenger (or experimenter) walking through the same area can be added into the database as an update. This is important as the position of large metallic objects may change over time. We have enabled passengers using the location app on their phones to also record and send us the refreshed calibration data as well. Of course, these measurements from thousands of passengers will give much more accurate results than our original calibration run which will soon be superseded by the new data.

4) Pedestrian Dead-reckoning mapping support

Steps are usually counted by detecting the specific combination of peaks and valleys from the waveform output of the accelerometer [10], [11]. The Fig. 3 shows the data of acceleration from the accelerometer in the cell phone's natural coordinate.

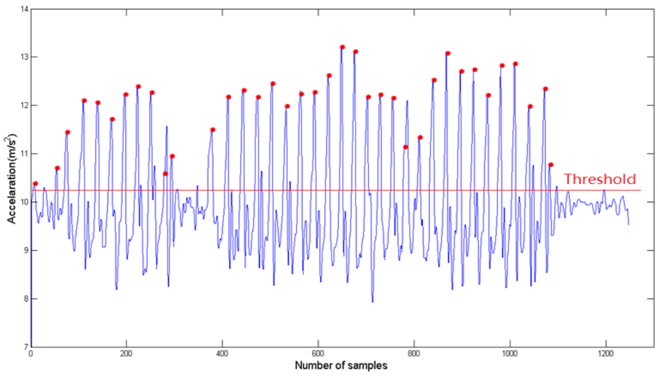

Fig. 3. Waveform from accelerometer when doing 35 steps walking

The red points indicate the maximum peaks of the acceleration. The system detects valid peaks and recognises the special pattern which could be deemed as a step. This pattern is a pair of peaks and valleys.

The magnitude difference between an effective pair of maximum and minimum should be larger than a pre-defined threshold to eliminate miscounting due to shaking. The threshold value can be obtained in the offline training stage. The time difference for an effective pair of peaks and valleys should be larger than a threshold which is set to be 250 ms. This setting refers to the Olympic record [12] about the peak stride frequency human can reach, which is to prevent a situation when a peak is followed by more than one consecutive valley, since otherwise extra steps would be counted. Moreover, the system adopts a fixed stride length which can be set by experimenters before each calibration. The sample rate of the accelerometer is 50 ms and 64 samples are proceeded as a time window to avoid transient errors.

Three sets of tests were carried out to evaluate the performance of the step counting algorithm in three different areas of the subway station. Experimenters held the cell phone and then moved at constant speed for 100 m and recorded the number of actual steps, measured steps and null steps respectively. Five tests were performed in each testing set to obtain the average result. The average stride length of the experimenter was 63 cm which could be used to roughly calculate the average error. Table I shows that the accumulated error is below 2 metres in a 100 steps test. To reach this accuracy, having a fixed stride length is a limitation of this method. However, for the mapping work, all parameters can be optimised by the experimenter to guarantee the accuracy.

TABLE I. THE RESULT OF 3 SETS OF TESTS FOR STEP COUNTING ALGORITHM

Testing set number	***1***	***2***	***3***
Actual steps	100	100	100
Average measured steps	97	98	103
Null steps	3	2	4
Average accumulated error	1.89 metres	1.26 metres	1.89 metres
Overall accuracy	97.3%		

B. Matching Algorithms

During the online matching phase, based on the idea of the K-Nearest Neighbours (KNN) matching algorithm [13], we designed a three-step matching method to select the k nearest neighbours (reference points) of the matching point, which is shown in Fig. 4.

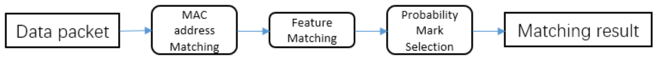

Fig. 4. Matching process

According to our experimental data, even when we set the data recording frequency to 0.25 seconds, the real tracking data packet was collected every 0.5 seconds on average. So, if we try to realise a real-time system with an updating frequency less than one second, we can only use every signal tracking data packet, which means that there is only one RSS value from each AP that can be utilised to matching with the database.

1) MAC address matching

The first step of the matching process is a method purely based on MAC address. The MAC address list L_d of tracking data packets is utilised to quickly select sub-databases for future precise matching. We design our database following the topology in Fig. 5.

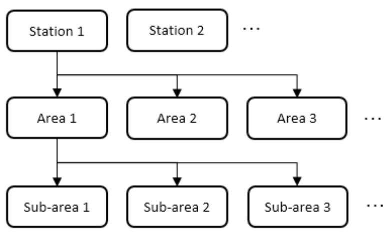

Fig. 5. Database topology

For each Sub-database of each Sub-area, we generate a MAC address list called impossible appearance list $L_i = L_f - L_s = [MAC_{m-n}, MAC_{m-n+1}, ..., MAC_m]$, where $L_f = [MAC_1, MAC_2, ..., MAC_m]$ is the MAC list of the upper area to which the sub-area belongs to; $L_s = [MAC_1, MAC_2, ..., MAC_n]$ is the MAC list of the sub-database. So, if the L_d fulfils the condition $L_d \cap L_i = \varnothing$, then the current position should be in the area covered by this sub-database.

In programming logic, if any single MAC address MAC_d in the tracking data packet matches with L_i : $MAC_d \in L_i$ we can determine that its current position is impossible within sub-database of corresponding area and start to scan the next sub-database. In other words, we do not have to traverse the whole database and quickly select a few sub-databases to do precise matching. Moreover, in a real environment experiment, data detected from all APs including those from personal cell phones, laptops, etc. will have no effect by this method.

2) Feature Matching

As we mentioned before, the RSS feature distribution is unique. So, it is unreasonable to use statistical characteristics e.g. mean value and standard deviation as well as traditional Euclidean distance based algorithms to match the tracking data with the database.

Assuming that a single tracking data RSS detected from one AP is s_x and the feature distribution of the AP at one reference point is $A = (a_0, a_1, ..., a_k)$, where a_0 to a_k are the RSS values of the distribution bins, we calculate $A' = |s_x - A| = (a_0', a_1', ..., a_k')$. For all RSS distributions at one reference point, we record all feature matched probabilities, Pr of matched APs, where $Pr = p_i \mid a_i' \le BW/2,\ i \in (0,k)$; p_i is a_i corresponded probability in feature histogram distribution. After counting the number of feature matched APs, if we have

$$\frac{Number\ of\ feature\ matched\ AP}{Number\ of\ detectable\ AP} \ge 70\%$$

, then the reference point has been successfully matched. In the following stage, all feature matched reference points will be evaluated by the Probability Mark Selection.

3) Probability Mark Selection

Assuming that the number of reference points has been selected in the previous stage is, *n*, we calculate a *Mark* for each of them to evaluate the precisely matching probability. Here we define:

$$Mark = \frac{\sum_{i=0}^{n} \mathrm{Pr}_i \cdot R_i \cdot D}{n}$$

where Pr_i is the feature matched probability; R_i is the probability of occurrence; D is the direction information coefficient. Here $D = 1.2$ gives 120% confidence of the Mark when the direction information has been matched, which gives our system the best performance. If the direction information does not match, then $D = 1$. For all calculated Marks, based on a *k*-nearest neighbours selection function

$$\sum_{i=0}^{k} Mark_i \ge 5\% \cdot \sum_{i=0}^{n} Mark_i, k \le n$$

the k-nearest neighbour reference points will be selected.

Finally, the weighted average coordinate of the selected k nearest neighbours will be calculated according to Mark of each selected result.

C. Discrete Kalman Filter Implementation

After online matching process, a discrete Kalman filter has been implemented to improve the tracking result accuracy and as a trajectory smoother. In the areas of position tracking and data prediction, the Kalman filter [14] is a widely used signal processing technique for trajectory optimisation. It is a two-step algorithm for linear systems to help improve the estimation of their next states. Based on a given system's current state and a dynamic model of the target trajectory, it initially predicts the next state of the system. The next step combines the predicted value with the actual value, giving rise to a more accurate estimate of the next state. More importantly, it takes into account observation noise and process noise when predicting the trajectories.

The Kalman filter model assumes that the true state s_t evolves from the previous state s_{t-1} according to the linear stochastic difference equation: $s_t = As_{t-1} + Bu_t + w_t$, where A is the state transition matrix, which relates the previous state at time *(t-1)* to the current state at time *t*. *B* is the control input matrix, which relates the optional control input vector u_t to the state s_t . w_t is the process noise which we assumed as $w_t \sim N(0,Q)$, $Q = \sigma_w^2$.

1) Prediction stage

During the prediction phase, the Kalman filter uses the previous optimal state estimate to produce a new estimate of the current state. That is known as discrete Kalman filter time update equations.

$$\hat{x}|_t = A\hat{x}|_{t-1} + B\hat{u}$$
$$P|_t = AP|_{t-1}A' + Q$$

P is error covariance; *t* represents state time; *t-1* is previous state.

2) Correction stage

During the correction phase, the Kalman filter corrects the estimate which is obtained during the prediction phase by comparing it with the observing result.

$$\hat{x}|_{t+1} = \hat{x}|_t + G|_t(z - \hat{x}|_t)$$
$$P|_{t+1} = P|_t(1 - G|_t)$$

where $G(t)$ is the Kalman gain with $G|_t = \dfrac{P|_t}{P|_t + R}$ and R is observation noise. Observation result, z, includes two input $z = [p, v]'$, where p is the matching result coordinate and v is the velocity provided by the PDR.

One point to declare is that we did not apply the filter on altitude change in our system because in most case the passengers are walking on flat ground and when the passenger is taking the escalator, there is no steps can be detected. Moreover, according to the experimental result, the filter leads to a larger error distance if the filter is applied to the altitude change. So, the filter is only applied to flat movement.

IV. Experimental Result

This section presents our experimental results in two subway stations. All experiments concentrated on the parts of the station where people transit regularly. We video recorded the experimental process so that the experimental results can be compared with ground truth along time point.

Experiment environment and equipment:

- Most of the subway station environments were narrow tunnels and corridors.
- Experiments were basically conducted in two scenarios of environment: 1) off-peak time with less than 10 people per minute coming and going on one platform. 2) peak time with approximately 140 people per minute coming and going on one platform. Both scenarios have situations of trains arriving and departing.
- Module of experimental standard cell phone: Samsung Galaxy s4.

We carried out experiments in different areas of the subway station. Fig.6, 7 and 8 are three examples of error distance distribution (by time) comparisons between peak time and off-peak time on the platform, tunnel and escalator, respectively. The average error distance of these three experiments is shown in Table II.

TABLE II. AVERAGE ERROR DISTANCE IN DIFFERENT AREAS

Experiment area	***Off- peak time***		***Peak time***	
	Matching result	***Filtered result***	***Matching result***	***Filtered result***
Platform	1.97 m	1.50 m	4.91 m	2.62 m
Tunnel	1.92 m	1.72 m	2.15 m	1.79 m
Escalator up	1.23 m	1.23 m	4.23 m	3.55 m
Escalator down	0.91 m	1.18 m	2.73 m	2.66 m

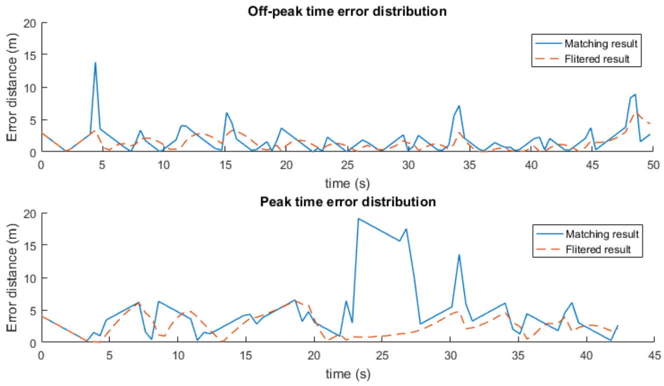

Fig. 6. Error distance distribution on the platform

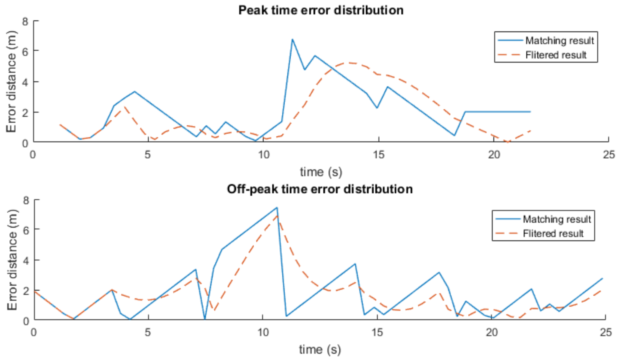

Fig. 7. Error distribution in tunnel

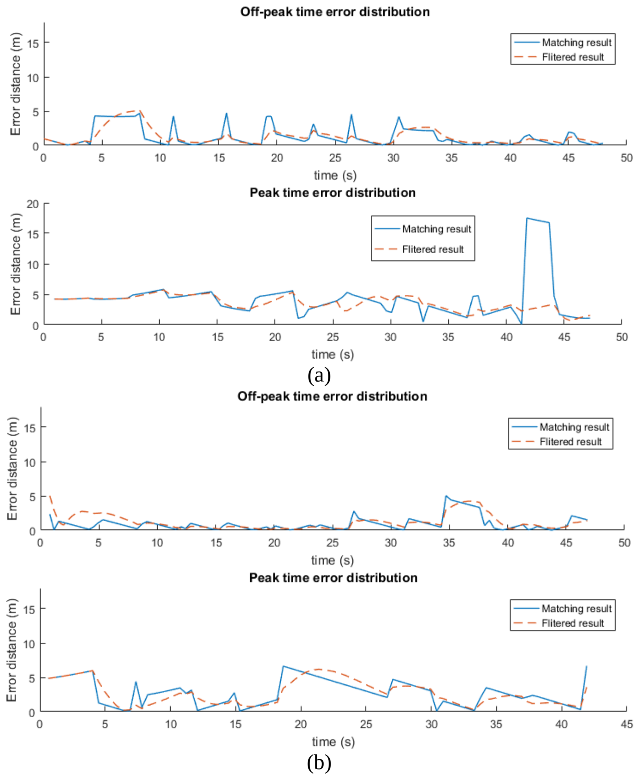

Fig. 8. Error distribution on (a) escalator up (b) escalator down

On the platform during peak time, the average error distance of matching result is 4.99 m. However, the filter is capable to correct the error, especially for those larger than 5 m, which reduce the average error to 2.5 m. The average error in narrow tunnel in both peak (1.79 m) and off-peak (1.72 m) situations achieved satisfied accuracy within 2 metres. As for the escalator, the filter has not provided significant improvement since the filter is not applied to the altitude change. However, both matching and filtered results are around 1 metre which is good enough during off-peak time; the matching results during peak time are close to the performance on the platform.

Fig. 9 and Table III presents the overall average error distance of experimental results from multiple experiments in all areas. Also, the 80% error Cumulative Distribution Function (CDF) indicates that even during peak-time in the station, the system still can provide a reliable position. Moreover, the Kalman filter gives a 0.5 metre improvement in accuracy. Compared with reported fingerprinting IPS accuracy in different scenarios [4], the performance of our system has reached accuracy within 2 metres in corridors and within 6 metres in indoor open spaces and corridors.

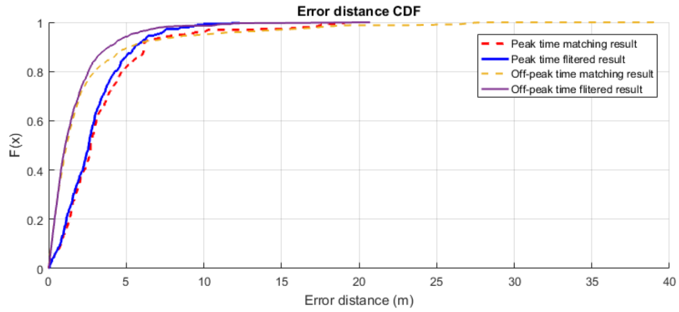

Fig. 9. Error distance CDF of off-peak time tracking and peak time tracking

TABLE III. SYSTEM PERFORMANCE COMPARISON DURING PEAK AND OFF-PEAK TIME IN THE STATION

System performance	***Off- peak time***		***Peak time***	
	Matching result	***Filtered result***	***Matching result***	***Filtered result***
Overall average error distance	2.37 m	**1.71 m**	3.42 m	**2.90 m**
80% CDF error distance	< 2.91 m	< 2.54 m	<4.77 m	<4.2 m

V. CONCLUSION AND FUTURE WORK

A real-time indoor positioning system for cell phones in two subway stations has been successfully implemented and demonstrated, which uses novel simple calibration and matching methods. The tracking accuracy, 1.7 metres during off-peak times and 2.9 metres during peak passenger traffic times, fully meets the initial aim of this research - to record the walking tracks of the passengers. For future work, once such a system has been initialised, all collected real passenger walking data can be added to the initial database by a similar method of calibration, which will keep the database continuously up-to-date.

ACKNOWLEDGMENT

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